(2) turns to be,

$$(2) = \frac{e^{-\frac{x^2}{4\alpha^2 t}}}{2\pi} \int_{-\infty}^{\infty} e^{-\eta^2} \cdot \frac{d\eta}{\alpha\sqrt{t}}$$

$$= \frac{e^{-\frac{x^2}{4\alpha^2 t}}}{2\pi\alpha\sqrt{t}} \int_{-\infty}^{\infty} e^{-\eta^2} d\eta \quad \text{———} \quad (3)$$

$$\int_{-\infty}^{\infty} e^{-\eta^2} d\eta = \left[\int_{-\infty}^{\infty} e^{-x^2} dx \cdot \int_{-\infty}^{\infty} e^{-y^2} dy \right]^{1/2}$$

$$= \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx\,dy \right]^{1/2}$$

$$= \left[\int_0^{2\pi}\int_0^{\infty} e^{-r^2} \underbrace{r\,dr\,d\theta}_{\frac{1}{2}dr^2} \right]^{1/2}$$

8.4. FT & $\mathcal{L}$T.

8.6 Numerical Sol.

$$= \left[\frac{1}{2}\int_0^{2\pi}\int_0^{\infty} e^{-r^2}\, d(r^2)\, d\theta\right]^{1/2}$$

$$= \left[\frac{1}{2}\int_0^{2\pi}\left(-e^{-r^2}\Big|_0^{\infty}\right) d\theta\right]^{1/2} = \left[\frac{1}{2}\int_0^{2\pi} d\theta\right]^{1/2}$$

$= \sqrt{\pi}$. Thus, (3) leads to

$$g(x) = \frac{1}{2\alpha\sqrt{\pi t}}\, e^{-\frac{x^2}{4\alpha^2 t}} \qquad \text{—— (4)}$$

Finally,

$$u(x,t) = f * g(x) = \int_{-\infty}^{\infty} f(\xi)\, g(x-\xi)\, d\xi$$

$$= \int_{-\infty}^{\infty} f(x-\xi)\, g(\xi)\, d\xi = \frac{1}{2\alpha\sqrt{\pi t}}\int_{-\infty}^{\infty} f(\xi) \cdot e^{-\frac{(x-\xi)^2}{4\alpha^2 t}}\, d\xi = \frac{1}{2\alpha\sqrt{\pi t}}\int_{-\infty}^{\infty} f(x-\xi)\cdot e^{-\frac{\xi^2}{4\alpha^2 t}}\, d\xi$$

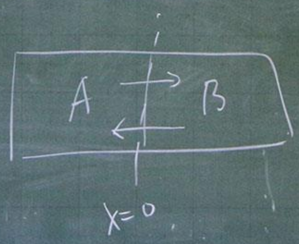
#

$$u(x,t) = \frac{1}{2\alpha\sqrt{\pi t}} \int_{-\infty}^{\infty} f(\xi)\, e^{-\frac{(x-\xi)^2}{4\alpha^2 t}}\, d\xi$$

$$= \frac{1}{2\alpha\sqrt{\pi t}} \int_{-\infty}^{\infty} \underbrace{f(x-\xi)}_{\text{unknown}} \cdot \underbrace{e^{-\frac{\xi^2}{4\alpha^2 t}}}_{\text{known!}}\, d\xi \qquad \#$$

Comments (x4)

C1.



$$f(x) = \begin{cases} F, & x \geq 0 \\ 0, & x < 0 \end{cases} = F H(x)$$

$$\therefore u(x,t) = \frac{F}{2\alpha\sqrt{\pi t}} \int_{0}^{\infty} e^{-\frac{(x-\xi)^2}{4\alpha^2 t}}\, d\xi \qquad \text{———} (1)$$

Set $\eta = \dfrac{\xi - x}{2\alpha\sqrt{t}}$; $d\xi = 2\alpha\sqrt{t}\, d\eta$

ξ	0	∞
η	$\frac{-x}{2\alpha\sqrt{t}}$	∞

$$\therefore (1) = \frac{F}{\sqrt{\pi}} \int_{\frac{-x}{2\alpha\sqrt{t}}}^{\infty} e^{-\eta^2}\, d\eta$$

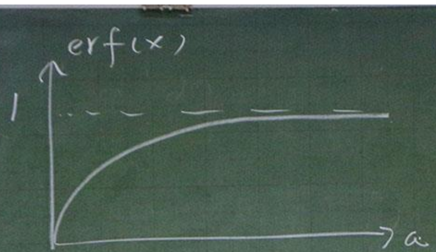
$$= \frac{F}{\sqrt{\pi}}\left[\int_{\frac{-x}{2\alpha\sqrt{t}}}^{0} e^{-\eta^2}\,d\eta + \int_0^\infty e^{-\eta^2}\,d\eta\right]$$

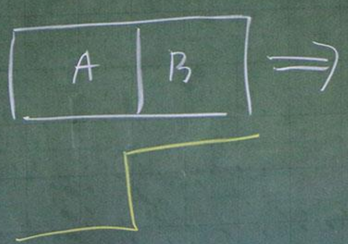
$$= \frac{F}{2}\left[\mathrm{erf}\left(\frac{x}{2\alpha\sqrt{t}}\right) + \mathrm{erf}\overset{\sqrt{\pi}/2}{(\infty)}\right] = \frac{F}{2}\left[1+\mathrm{erf}\left(\frac{x}{2\alpha\sqrt{t}}\right)\right]$$

, where $\underset{\text{Error}}{\underline{\mathrm{er}}}\mathrm{f}(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x e^{-\eta^2}\,d\eta \quad \therefore \mathrm{erf}(\infty) = 1$

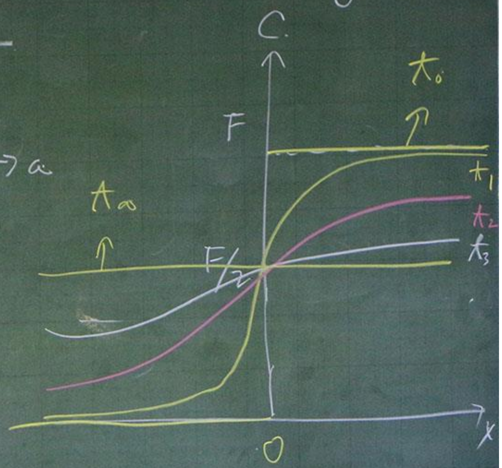
$+\mathrm{erf}\left(\frac{x}{2\alpha\sqrt{t}}\right)$

$\infty) = 1$

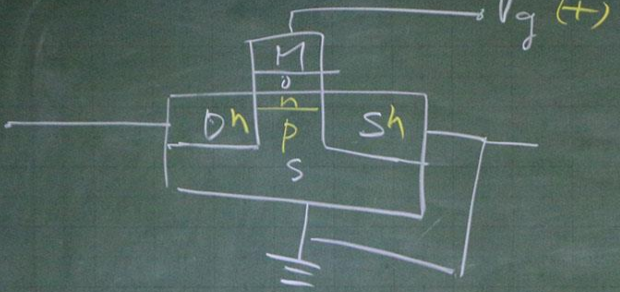




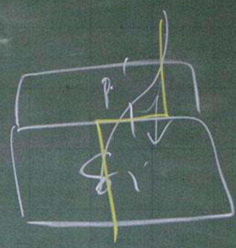
C.Z. "Smoothing"



MOSFET



drive in



c3.

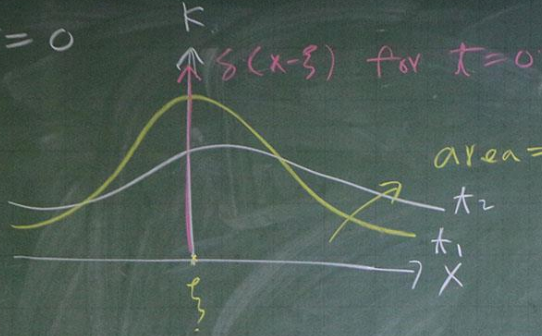
$$u(x,t) = \int_{-\infty}^{\infty} f(\xi)\, K(x-\xi, t)\, d\xi$$

① $$K \equiv \frac{1}{2\alpha\sqrt{\pi t}}\, e^{-(x-\xi)^2/4\alpha^2 t}$$

② K happens to a Gassian fxn

$$\frac{1}{2\alpha\sqrt{\pi t}} \int_{-\infty}^{\infty} e^{\overset{\eta^2}{-(x-\xi)^2/4\alpha^2 t}}\, d\xi = 1$$

③ $t = 0$



③. $t=0$, $K(x-\xi, t)$ is more focus, and its area remains 1 $\Rightarrow \lim_{t\to 0} K(x-\xi, t) = \delta(x-\xi)$.